Lecturer 4, 5

Pulse Code Modulation

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TIP
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Contents

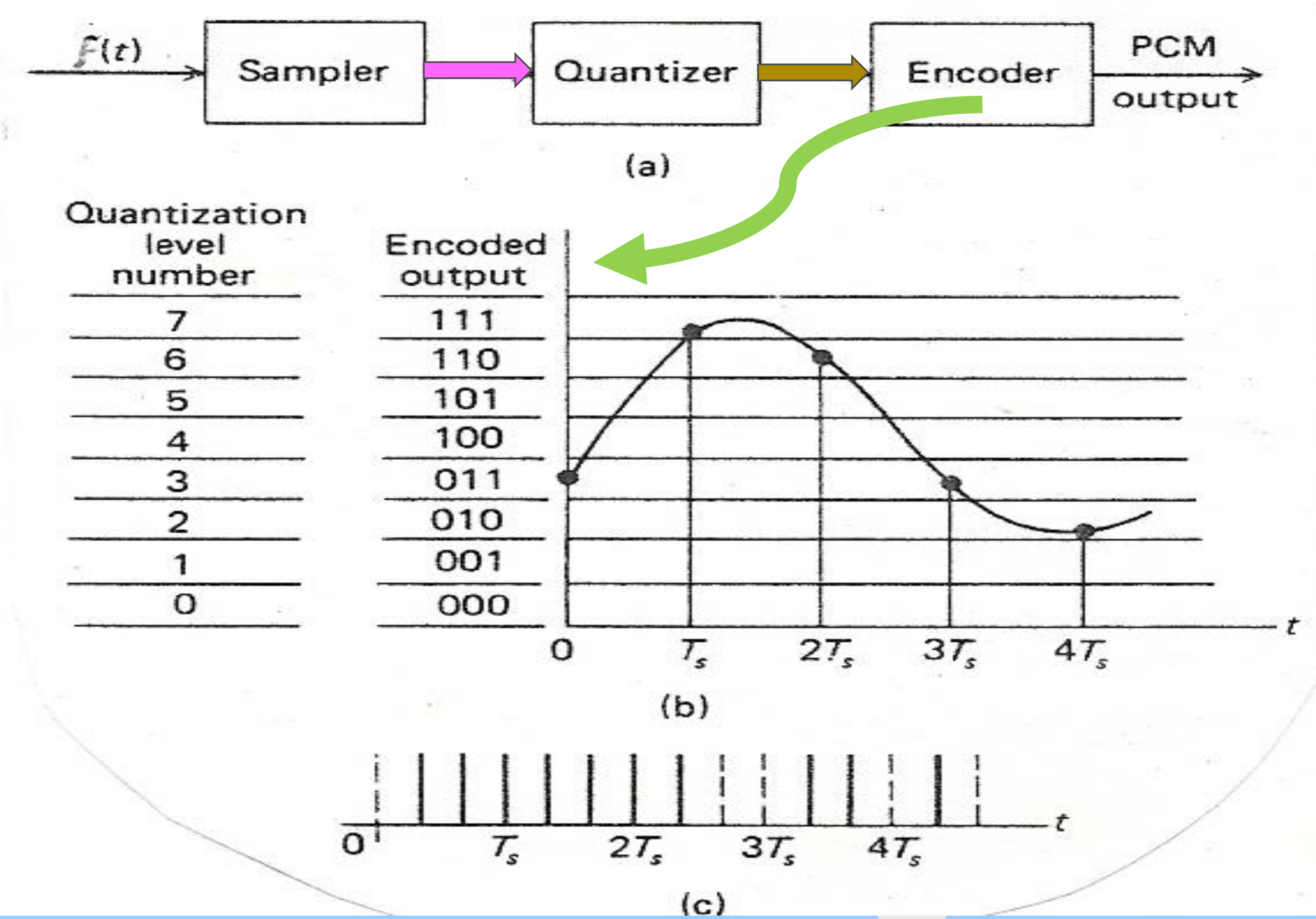
PCM

Generation of PCM

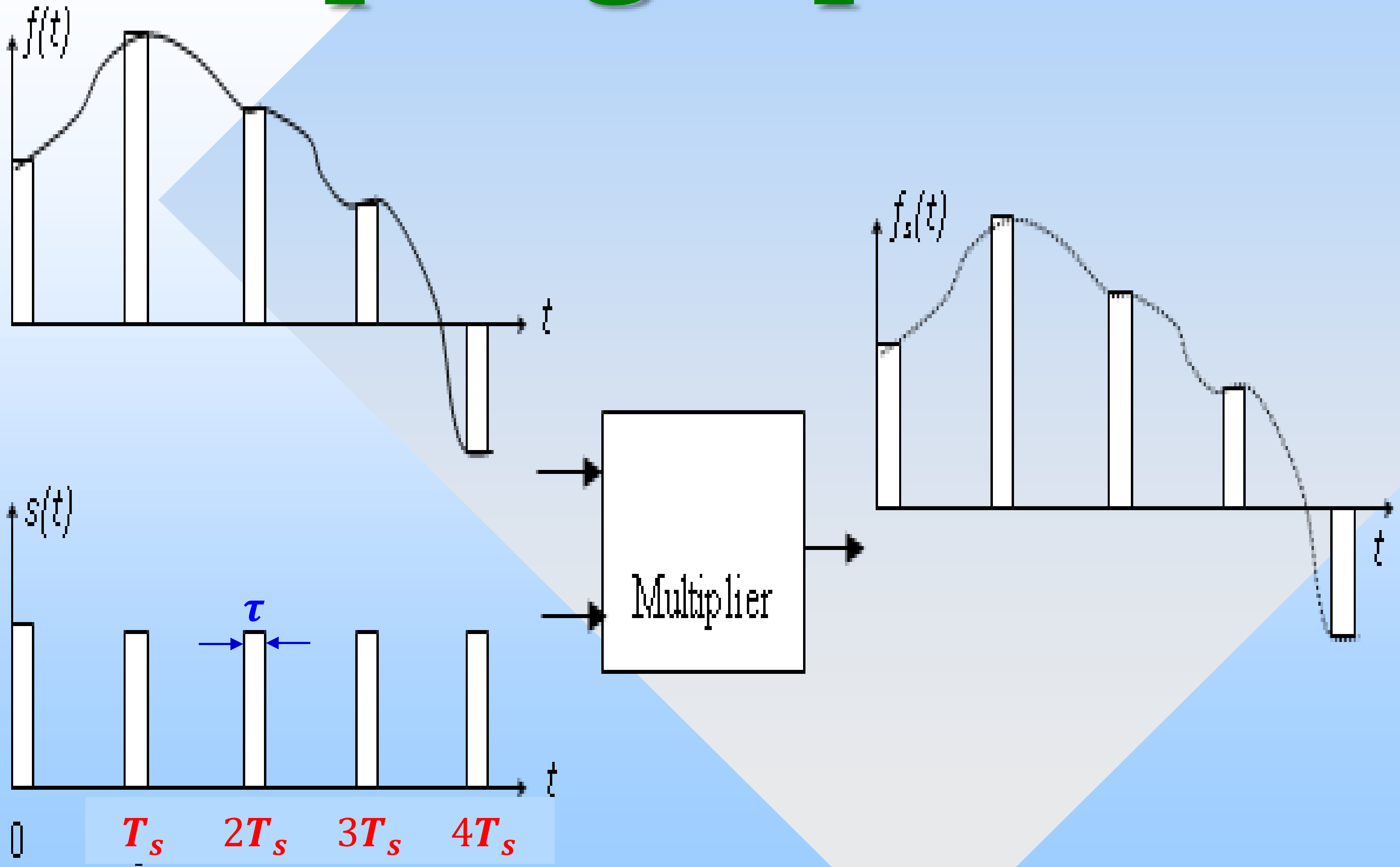
Consists of three processes:

- **Sampler:** Message signal $f(t)$ is first sampled by a rate $f_s > 2f_m$.
- **Quantizer:** Sample values are then quantized to a certain levels.
- **Encoder:** Quantization levels are encoded into binary sequence.

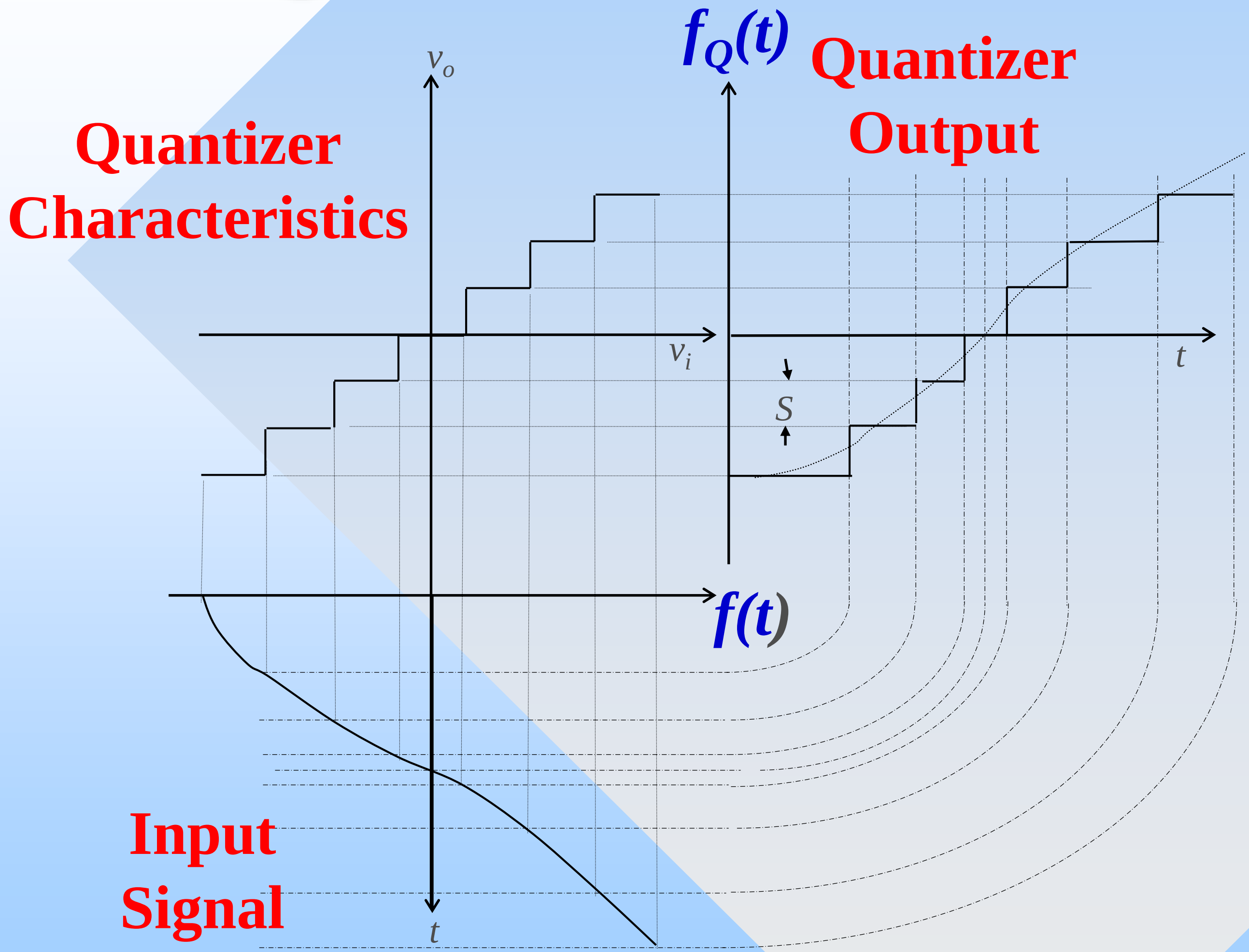
PCM Block Diagram



Sampling Operation



Quantization



Quantization Error

- Difference between original signal $f(t)$ and its quantized approximation $f_Q(t)$
- Why termed as quantization noise?
  - Affects the signal amplitude.
  - May be added to or subtracted from the signal.
  - Expected average value is zero.
- Maximum value is ½ least significant

$$Q_{maximunm} = \frac{v_{LSB}}{2}$$

- Upon reconstruction:
  - Added noise may be removed.
  - Sometimes errors take place.

$\overline{e^2} = \frac{S^2}{12}$

Quantization Error

- Mean square quantization error voltage at quantizer output can be shown to be:

$$\overline{e^2} = S^2/12$$

- Quantization error voltage is: $N_{ov} = \overline{e} = S/\sqrt{12}$
- Signal voltage at the quantizer output is:

$$S_{ov} = SM/2$$

- Signal to quantization error voltage ratio:

$$\frac{S_{ov}}{N_{ov}} = \sqrt{3}\ M$$

- Signal to quantization error power ratio :

$$\frac{S_v}{N_o} = 3\ M^2 = -4.8 + 20\ Log\ M\ dB$$

Quality of Quantization

- Quality of approximation improved by reducing the step size.
- Tests for speech indicate that:
  - 2 levels are understandable but quite noisy.
  - 8 or 16 levels are sufficient for a good intelligibility.
  - 128 or 256 are usually used to ensure high quality.
- Tests for color TV:
  - 64 levels gives only good color TV.
  - 512 levels is used for commercial color TV.

Missed Signal Details

- In quantizing some details are lost.
- It is impossible to reconstruct the original.
- However, there is no need to transmit all signal details:
  - Ears and eyes in hearing and watching is limited:
    - Ear could not distinguish small distortion.
    - Eye has a limited resolution.
  - Also, due to noise, detector will not able to distinguish fine variation.

Probability of Level Error

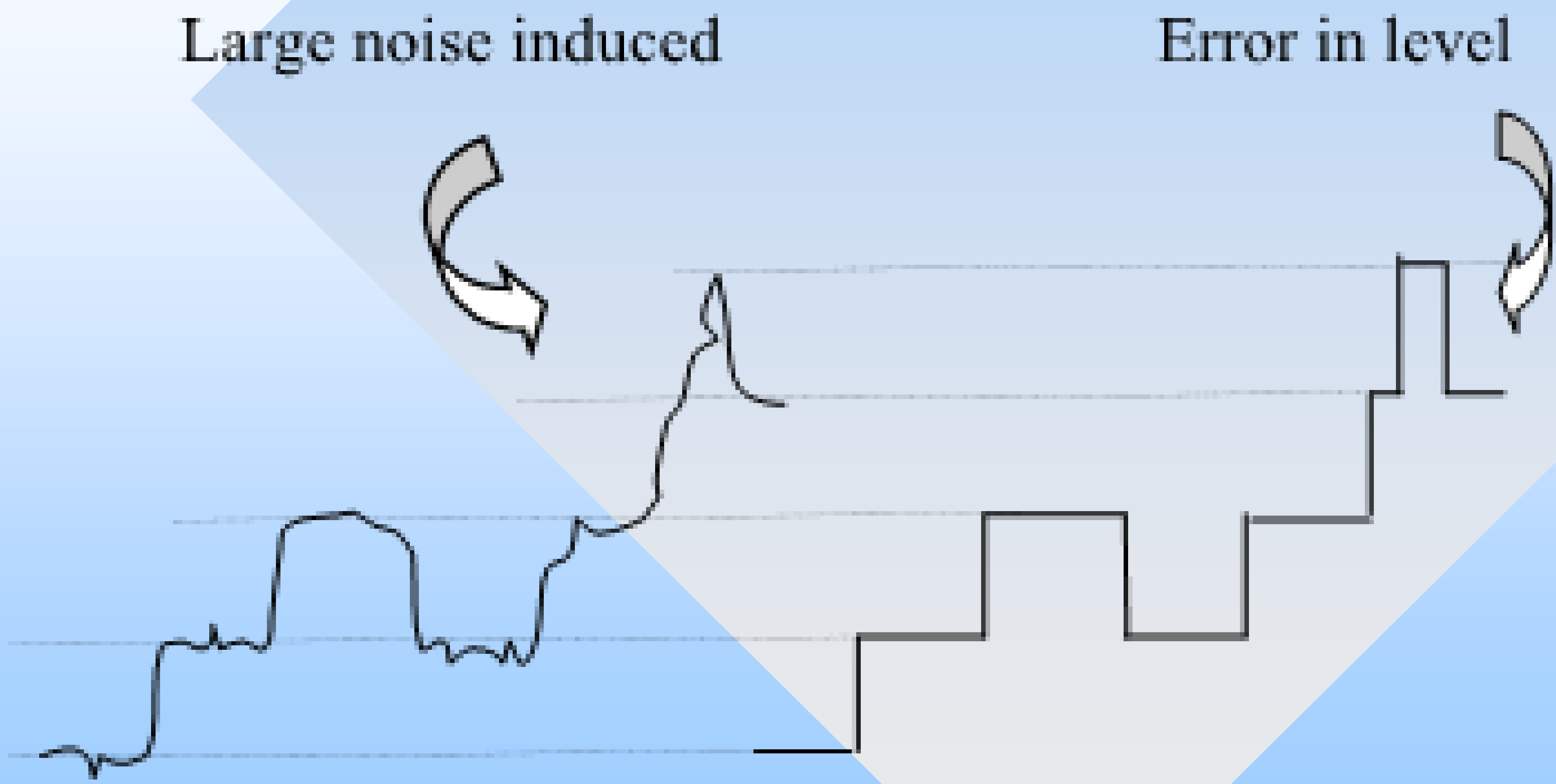


Fig.1.18: Noise Removing or Level Error

Encoding

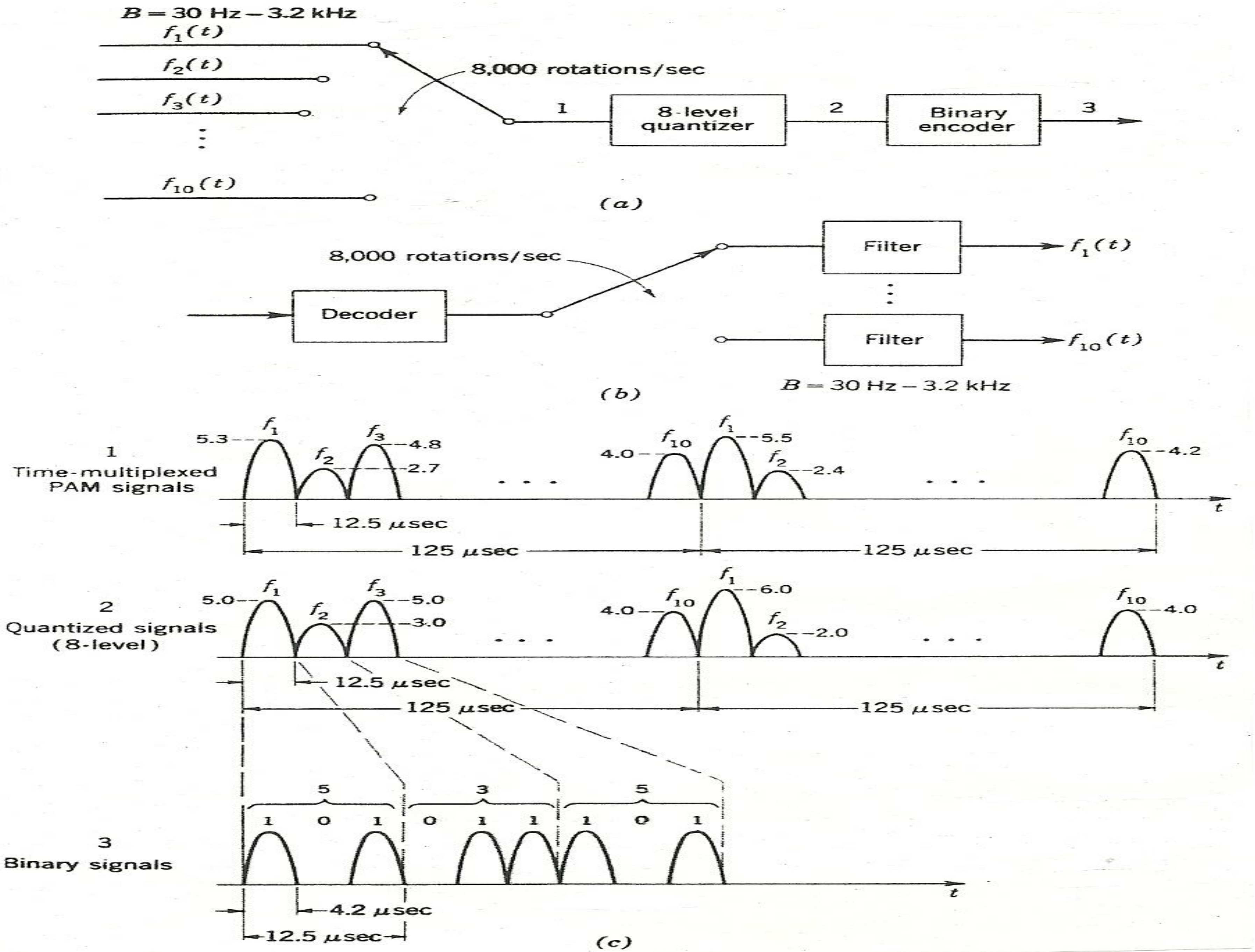
- A binary code where n equals 2.
- Number of quantization levels M is related to the number of bits per sample n as:

$$M = 2^n$$

- Generally, bandwidth for transmission of pulse train is inversely proportional to its width and depends on its shape.
- Therefore, bandwidth is roughly given by the reciprocal of the time slot.

Example

- Assume 10 channels PCM System
  - Sampled PAM,
  - Quantized, and
  - PCM using 8 levels quantizer.
- Required Bandwidths:
  - PAM signal bandwidth is 1/12.5 μsec = 80 kHz.
  - Quantized PAM signal bandwidth is 80 kHz.
  - PCM signal bandwidth is 1/ 4.2 μsec = 240 kHz.



B = 30 Hz – 3.2 kHz
f1(t)
f2(t)
f3(t)
f10(t)
8,000 rotations/sec
1
8-level quantizer
2
Binary encoder
3
(a)
8,000 rotations/sec
Decoder
Filter
f1(t)
Filter
f10(t)
B = 30 Hz – 3.2 kHz
(b)
1
Time-multiplexed PAM signals
5.3
f1
f2
f3
4.8
2.7
4.0
f10
f1
5.5
f2
2.4
f10
4.2
12.5 μsec
125 μsec
125 μsec
2
Quantized signals (8-level)
5.0
f1
f2
f3
5.0
3.0
4.0
f10
f1
6.0
f2
2.0
f10
4.0
12.5 μsec
125 μsec
125 μsec
5
3
5
1 0 1 0 1 1 1 0 1
3
Binary signals
4.2 μsec
12.5 μsec
t
(c)

Exercises

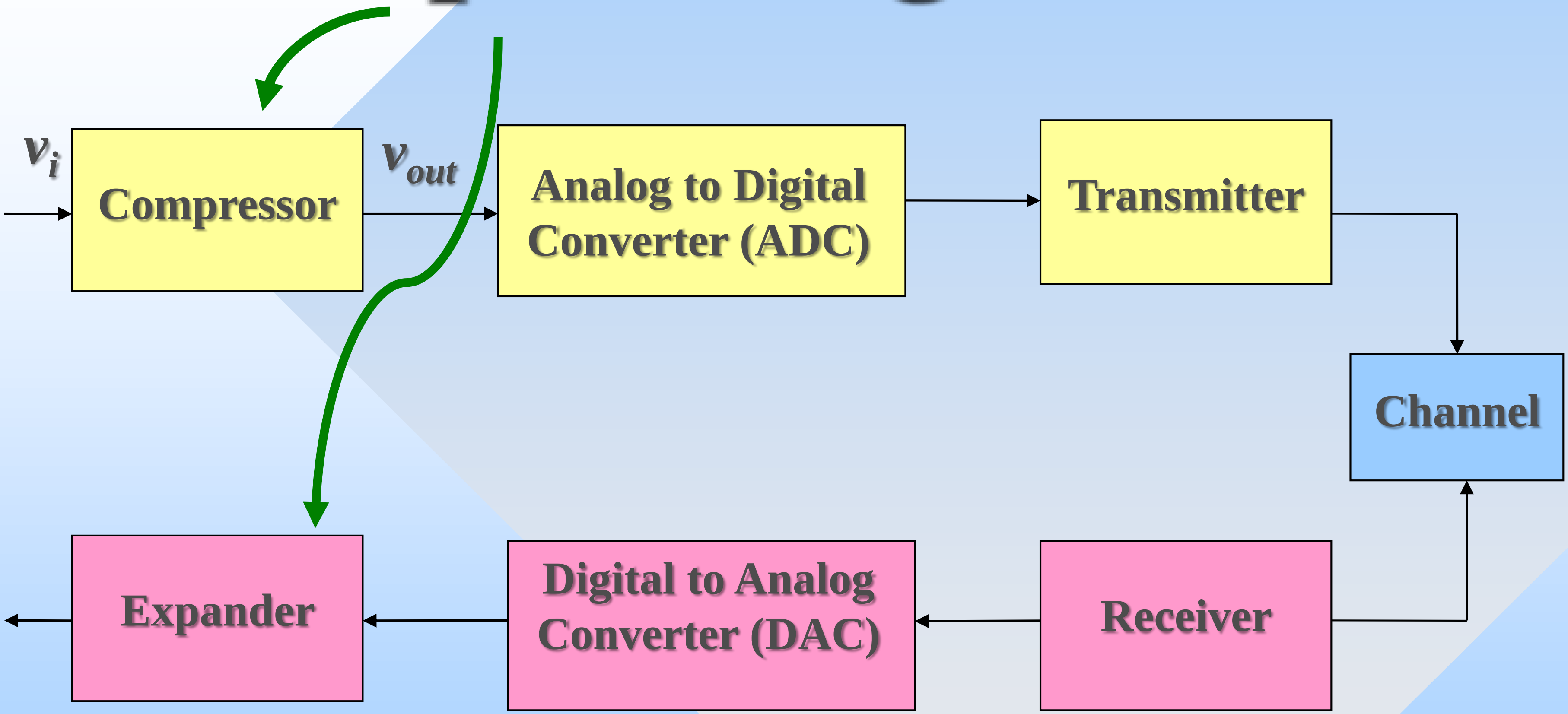
- Indicate the advantages of PCM systems when compared to PAM or Delta techniques.
- What is difference between the unipolar and the bipolar quantization?
- What is difference between mid-rise and mid-tread quantization processes?
- What is difference between the uniform and the nonuniform quantization?

Companding

Reason of Compading

- Small signals will have a poorer signal to quantization noise ratio SQR than large.
- So, it is better to have moderate levels for both low and high variations of the signal:
  - Increase low signal levels to more moderate levels so that SQR could be increased.
  - Reduce high signal levels to more moderate levels in order to decrease high SQR.

Companding Process



Companding Laws

- Companding means compression and expansion.
- Compression by using special designed diodes prior to sampling circuit
- Whereas expansion is attained with diodes after the receiver LPF.
- Voice signal require a constant SQR over a wide dynamic range DR. This requires a logarithmic compression ratio. There are two methods:
  - μ Law Companding.
  - A Law Companding.

μ Law Companding (USA and Japan)

μ Law Compression

$$v_o = \frac{\ln\left[\left\{1 + \mu\left(\frac{v_i}{V_{i,max}}\right)\right\}\right]}{\ln[1+\mu]} V_{o,max}$$

$V_{i,\max}$: Max amplitude of input signal before compression.

$V_{o,\max}$: Max amplitude of output signal after compression.

v_i: Amplitude of input signal before compression for $v_i > 0$

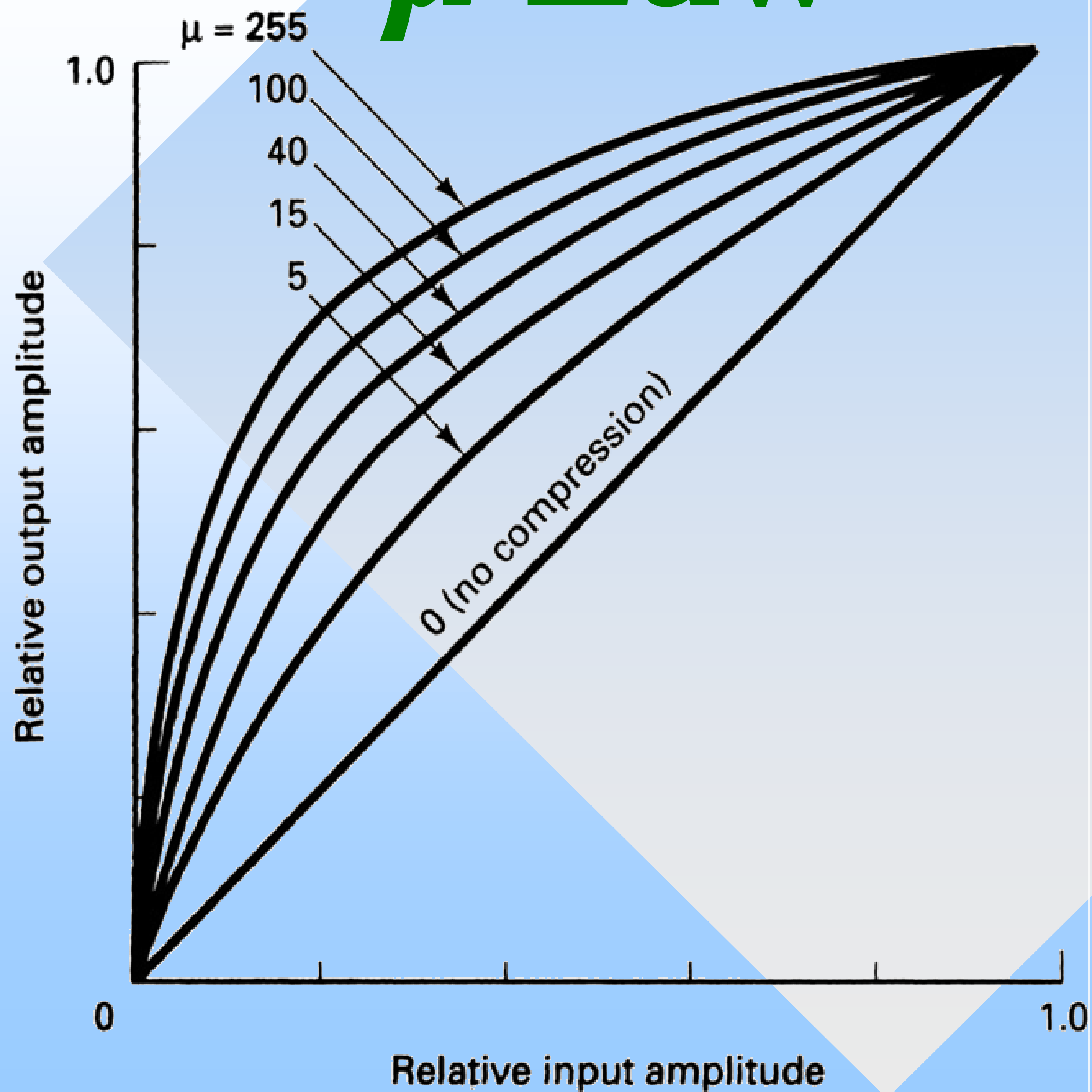
v_o: Amplitude of the output signal after compression.

μ: is a measure for the amount of compression.

μ: determines the range of signal power over which SQR is relatively constant.

- For DR = 40 dB, 7 bit PCM code uses μ = 100.
- For DR = 40 dB, 8 bit PCM code uses μ = 255.

μ Law



μ = 255
100
40
15
5
0 (no compression)
1.0
0
1.0
Relative output amplitude
Relative input amplitude

μ Law Expansion

On reception, received signal will be expanded to satisfy linearity. Henceforth, received signal is:

$$v_{o,r} = \frac{V_{i,r,max}}{\mu}\left[(1+\mu)\left(\frac{v_{i,r}}{V_{o,r,max}}\right) - 1\right] \quad , v_{i,r} \geq 0$$

$V_{i,r,\max}$: Max amplitude of input received signal before expansion.

$V_{o,r,\max}$: Max amplitude of output received signal after expansion.

$v_{i,r}$: Amplitude of input received signal before expansion.

$v_{o,r}$: Amplitude of the output received signal after expansion.

A Law Companding (Europe CCITT)

A Law Compression

Compression characteristics is a true logarithm

$$v_o = \frac{\frac{Av_i}{V_{i,max}}}{1+\ln A} V_{o,max}, \quad 0 \le \frac{v_i}{V_{i,max}} \le \frac{1}{A}$$

$$v_o = \frac{1 + \ln\left[\frac{Av_i}{V_{i,max}}\right]}{1 + \ln A} V_{o,max}, \qquad \frac{1}{A} \le \frac{v_i}{V_{i,max}} \le 1$$

$V_{i,\max}$: Max amplitude of input signal before compression.

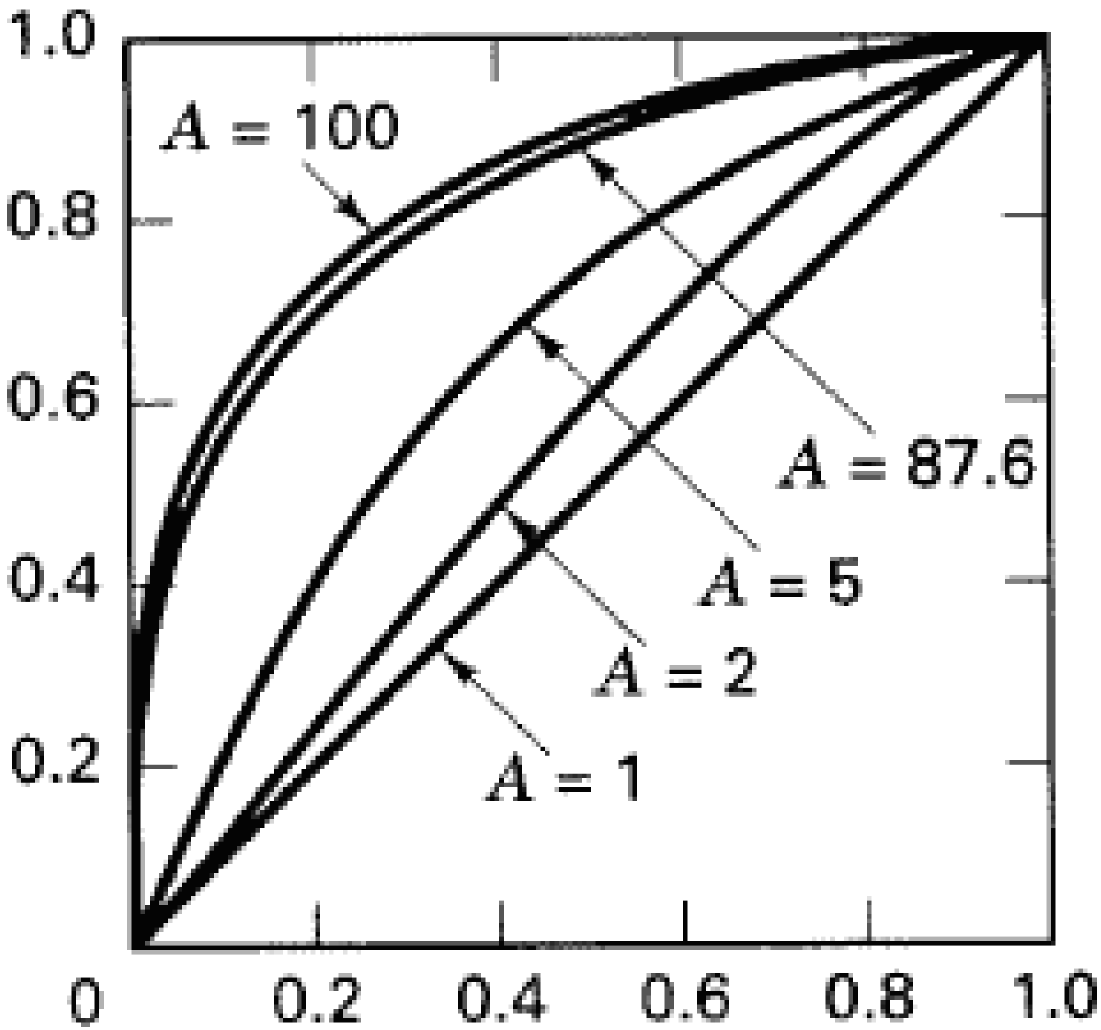
$V_{o,\max}$: Max amplitude of output signal after compression.

v_i: Amplitude of input signal before compression.

v_o: Amplitude of the output signal after compression.

Optimum value for voice transmission is $A = 87.6$.

A Law



Data Rate of PCM

- Speech signal for telephone has $f_m = 4$ kHz.
- So, the sampling rate is $2x4 = 8$ kHz, that is 8000 samples/sec.
- Sampling interval $1/8000 = 125\ \mu$ sec/Frame
- Each sample is encoded into 8 bits/sample.
- So, the data rate for PCM signal is:

$$R_{PCM} = (8000\ \text{samples/sec})(8\ \text{bits/sample})$$
$$= 64\ \text{k bits/sec}$$

North American PCM T1 Standard

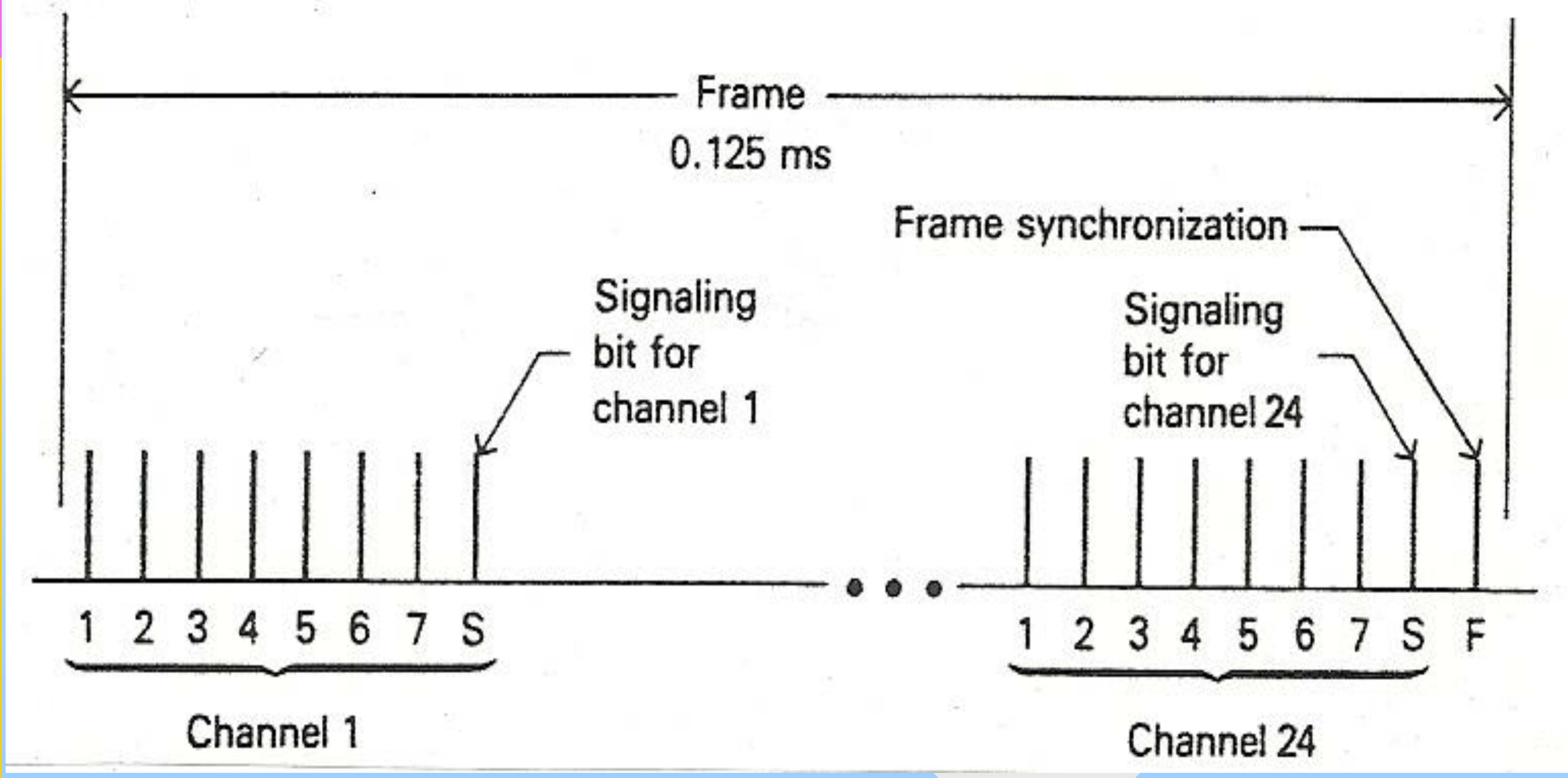
- Bell System in USA introduced 24 channel PCM in1960s for digital voice over short haul distances of 10 to 50 miles "T1".
- T1 has found widespread adoption in US, Canada, and Japan.

PCM T1 Standard

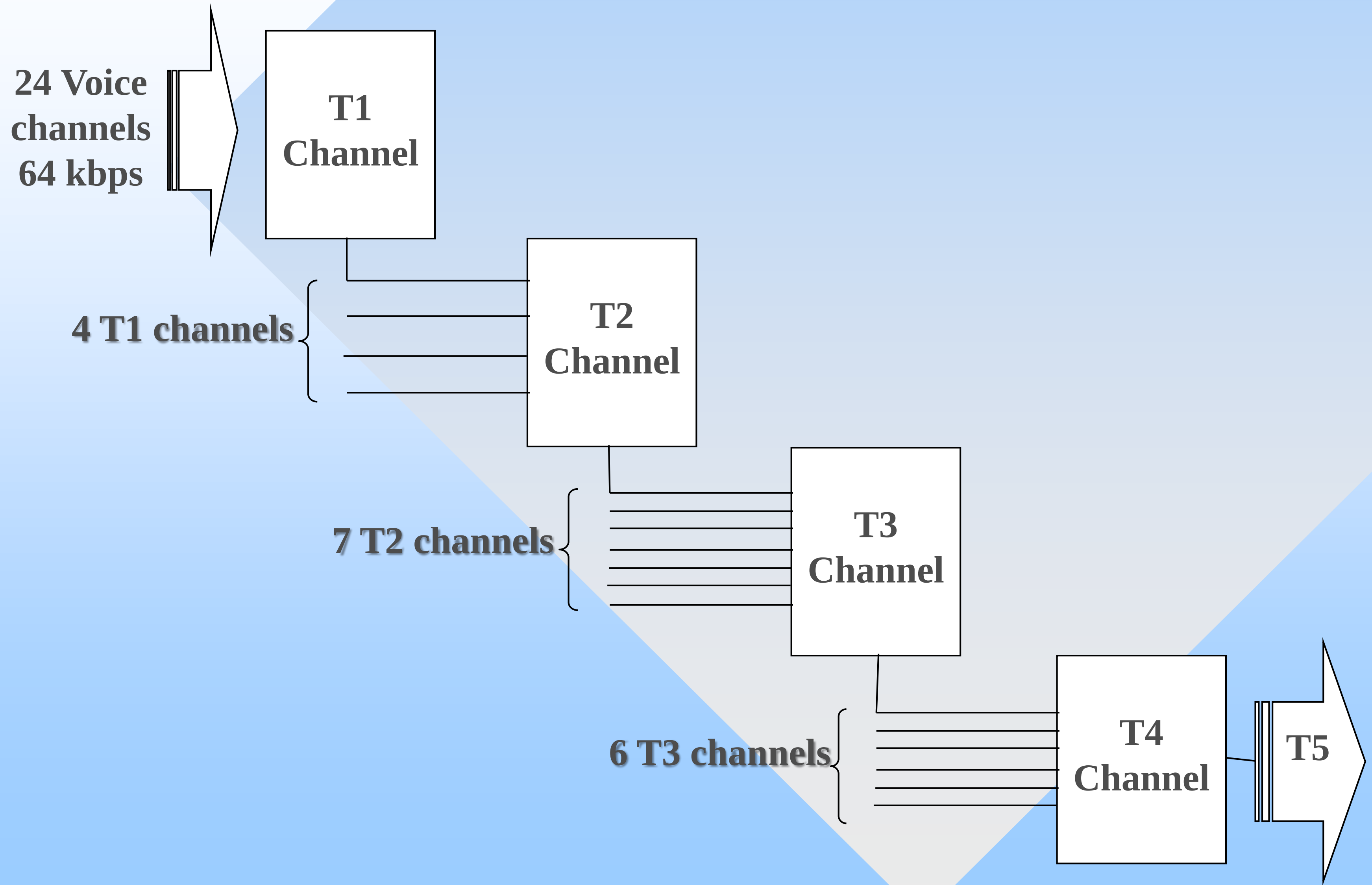
- Early, it uses $2^7 = 128$ quantization levels.
  - Each sample is quantized into 7 bits.
  - 1 bit for establishing calls (signaling).
- Recently, $2^8 = 256$ levels have been adopted for quieter system with less distortion.
- 24 channels are time multiplexed, sampled, and coded into 8 bit PCM formats in addition to 1 bit for frame synchronization.
- The frame consists of $24 \times 8 + 1 = 193$ bits.

$$R_{T1} = (193\ \text{bits/Frame})(1\text{Frame}/125\mu\text{sec}) = 1.544\ \text{Mbps}$$

North American PCM Standard for Short-Haul Telephone [T1 System]



Short to Long Haul PCM Systems



Exercises

Exercise 1.10

- Prove that the transmission data rate of T1 PCM system used in United States, Canada and Japan is given as 1.544 Mbps.

Exercise 1.11

- Estimate the number of channels and the data rate for long haul T2, T3, and T4.

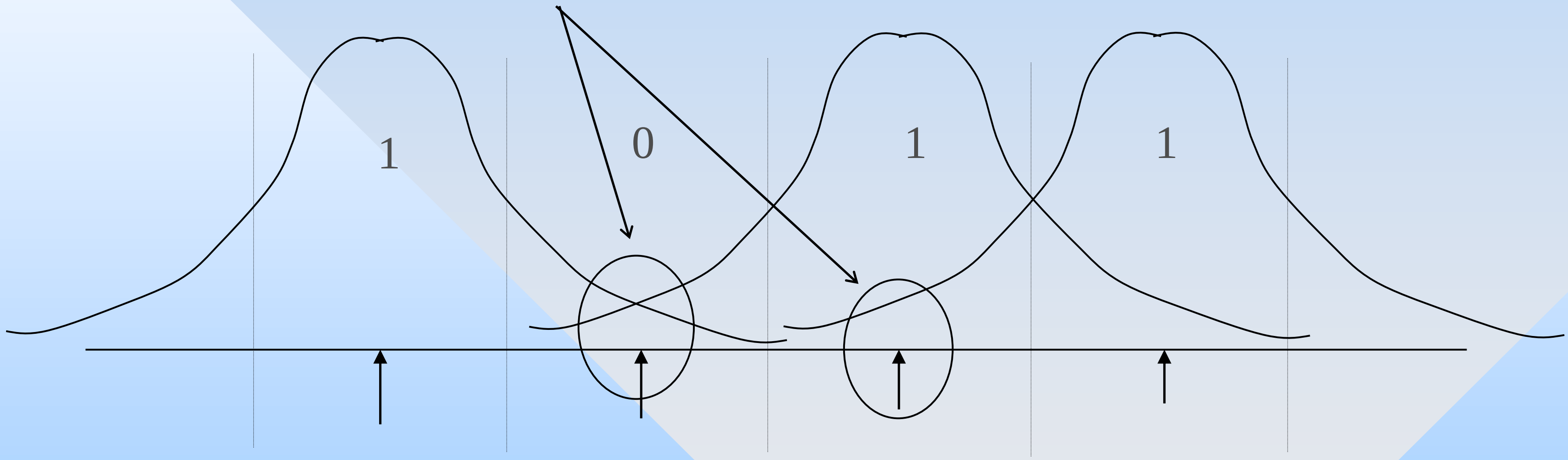
ISI

Intersymbol Interference ‘ISI’

- **First;** Bandwidth of PCM channel is restricted, waveforms will be distorted in a way analogous to crosstalk in PAM.
- **Second;** If digital pulses modulate a carrier for transmission over long distances, pulse shaping is a must.
- This causes pulses to spread out as they transverse channel and overlap into adjacent.

ISI

Inter-symbol Interference



Crosstalk and ISI

- In un-quantized PAM, adjacent <u>time slots</u> are often associated with different message, and crosstalk is appropriate.
- In PCM <u>adjacent bits</u> are generally symbols in the code representation of a <u>single quantized sample</u>, hence the term inter-symbol interference.

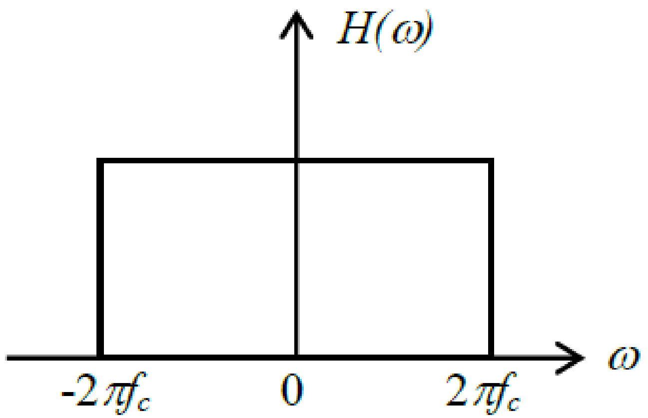
Minimizing ISI

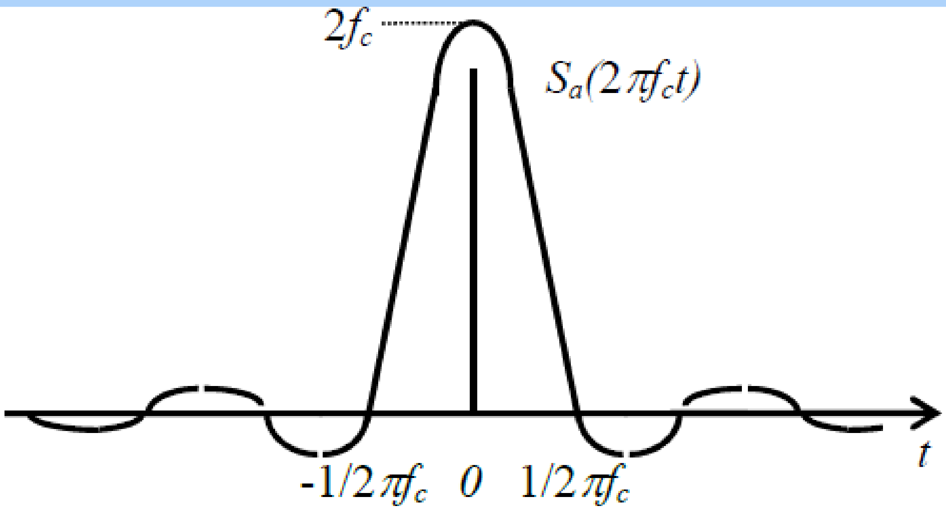
- Widening the transmission bandwidth as much as desired may minimize intersymbol interference.
- Instead, one could seek a way of purposely designing:
  - Signal wave-shapes and
  - Transmission filters used

Sampling Ch^s

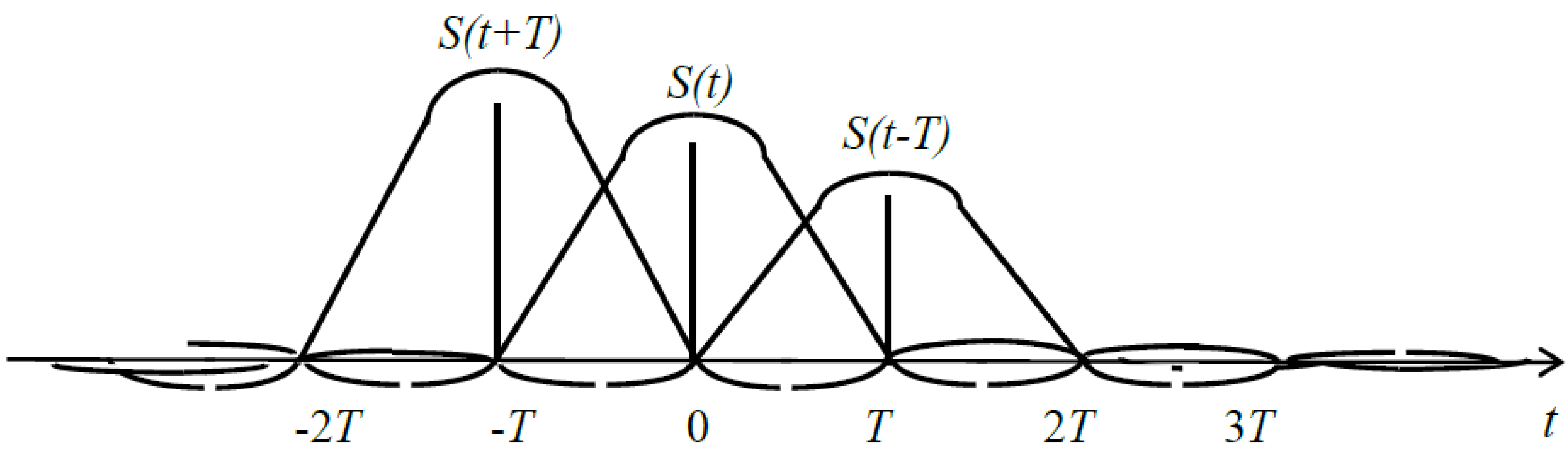
- One wave-shape producing zero ISI is the sampling function: $(sin 2\pi f_c t)/2\pi f_c t$
- Is impulse response of ideal LPF at f_c:
- Pulse goes zero at multiples of 1/2πfc
- If sample interval is chosen 1/2πfc adjacent pulses will not interfere

Pulse Providing Zero ISI





Sequence of Digital Pulses with Zero ISI



Practical Difficulties

- It implies the overall ch^s between the transmit and receive is ideal LPF.
- Require precise synchronization. If timing at receiver varies from exact synchronization, zero ISI condition disappears, the tails of all adjacent pulses may add up.
- Also some timing jitter will be present even with most sophisticated synchron.

Raised Cosine

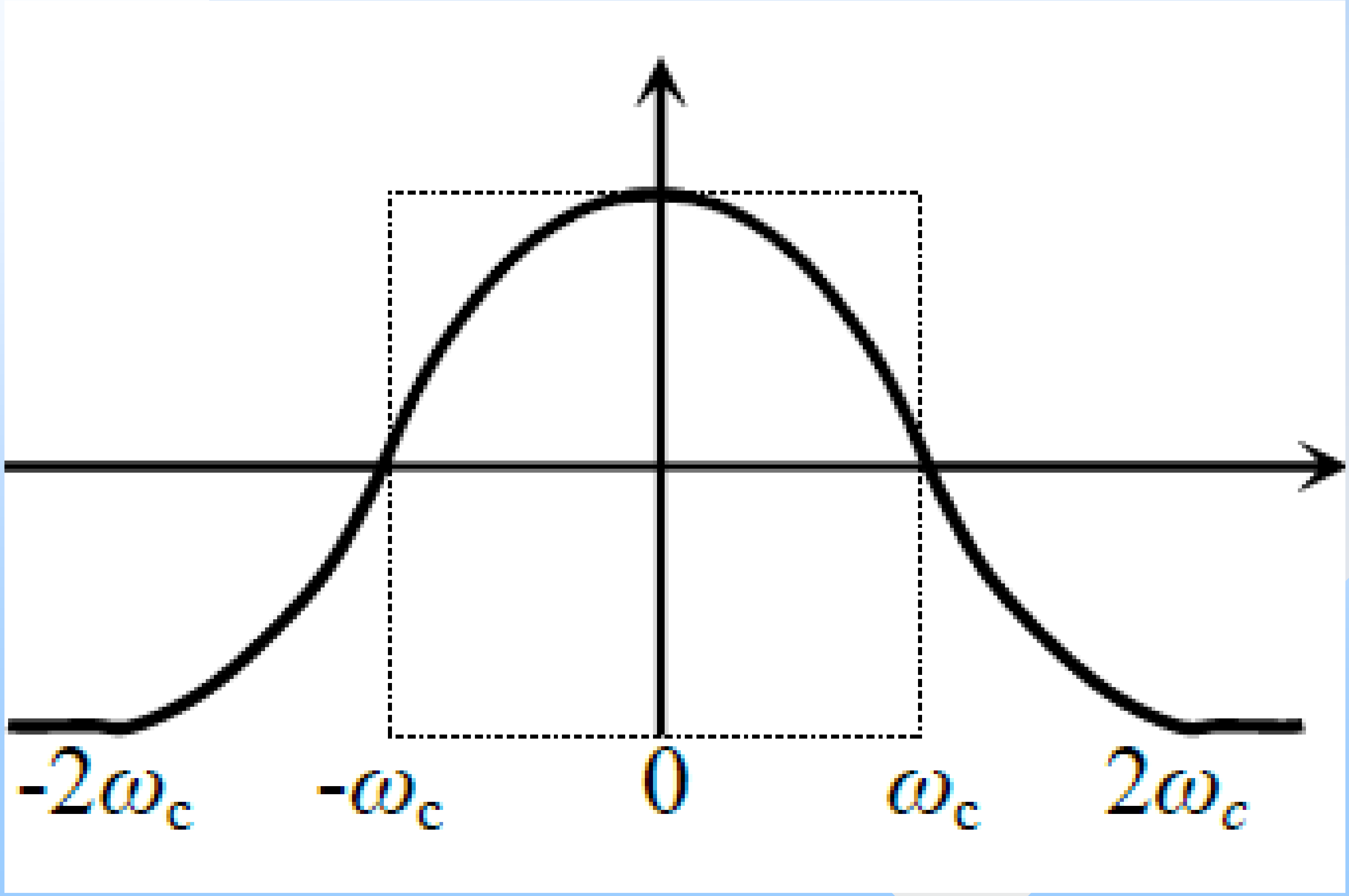
- If pulse has odd symmetry about LPF cutoff, impulse response retains property of zeros at uniformly spaced interval.
- An example is the raised cosine:

$$H(\omega) = \frac{1}{2}\left(1 + \cos\frac{\pi\omega}{2\omega_c}\right) \qquad \& \qquad \omega \le 2\omega_c$$
$$= 0 \qquad\qquad elsewhere$$

- To show symmetry, let $\omega = \omega_c + \Delta\omega$:

$$H(\omega) = \frac{1}{2}\left(1 + \cos\left\{\frac{\pi}{2} + \frac{\pi}{2}\frac{\Delta\omega}{\omega_c}\right\}\right) = \frac{1}{2}\left(1 - \sin\frac{\pi}{2}\frac{\Delta\omega}{\omega_c}\right)$$

Cosine Odd Symmetry



Impulse Response

- The impulse response of a filter with the characteristics shown is given by:

$$h(t)=\frac{\omega_c}{\pi}\frac{\sin\omega_c t}{\omega_c t}\left(\frac{\cos\omega_c t}{1-(2\omega_c t/\pi)^2}\right)$$

- It has the (sin x)/x term multiplied by an additional factor decreases with time.
  - (sin x)/x ensures zero crossing as for LPF.
  - Additional factor reduces the tails of pulses so that it becomes insensitive to timing jitter.

Digital Signaling Formats

Binary Line Coding

- Binary 1's and 0's may be represented in various signaling formats or line codes.
- The most popular are:
  - Unipolar Nonreturn to Zero, UNZ.
  - Unipolar return to Zero, URZ.
  - Bipolar Non return to Zero, PNZ.
  - Bipolar Return to Zero, PRZ.
  - Manchester

Desirable Properties of Line Codes

- •Self-synchronization.
- •Error detection capability:
- •Low probability of error:
- •Suitable spectrum for channel:
- •Transparency:

Self-synchronization

- There is enough timing information built into the code,
- So bit synchronizer can be designed to extract the clock signal.
- So, long series of binary 1's and 0's should not cause a problem in time recovery.

Error detection capability

The ability of addition of the channel encoders and decoders

Or incorporating them into the line code.

Low probability of error

- Receivers can be designed to recover data with low probability of error when the input is corrupted by
- Noise or
- Intersymbol interference ISI.

Suitable Spectrum for Channel

- With ac coupled, the power spectral density of line code should be negligible at frequencies near zero.

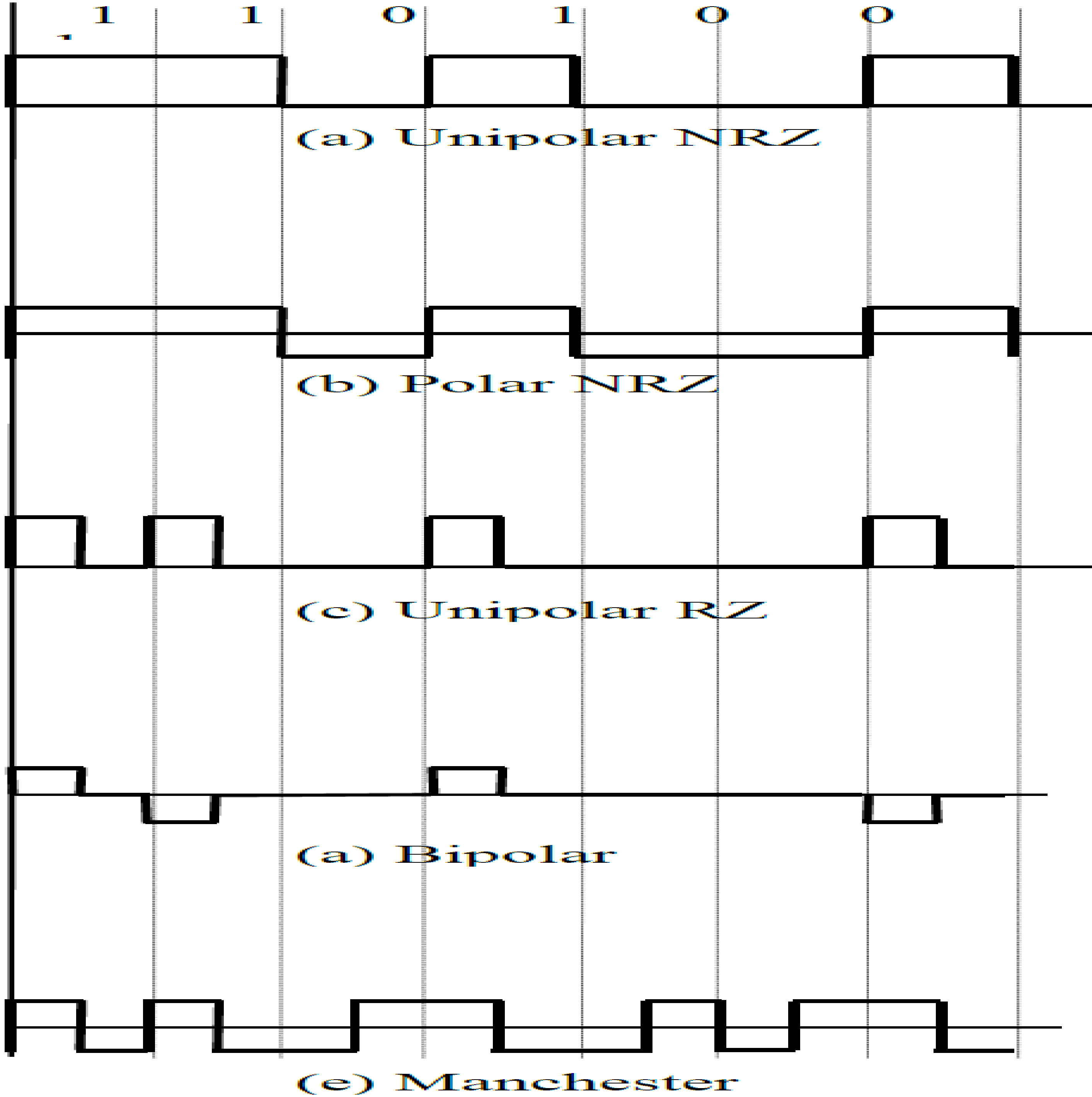
- Also, the signal bandwidth needs to be sufficiently small compared to channel bandwidth to minimize ISI.

Transparency

- Every possible sequence of data is faithfully and transparently received.
- A code is not transparent if some sequence will result in a loss of synchronization at the receiver.
- For example, the bipolar format is not transparent since a string of zeros will result in a loss of clocking signal.

Unipolar NRZ

- (on-off keying):
- The binary 1 is represented by a high level and
- a binary 0 by a zero level.
- High level does not return to zero during the binary 1 signaling interval.


1 1 0 1 0 0
(a) Unipolar NRZ
(b) Polar NRZ
(c) Unipolar RZ
(a) Bipolar
(e) Manchester

Polar NRZ

- Simply NRZ:
- Binary 1’s and 0’s are represented by equal
  - » positive
  - » and negative levels.

Unipolar RZ

- The binary 1 is represented by a high level over half of the bit period and then returns to zero.

Bipolar

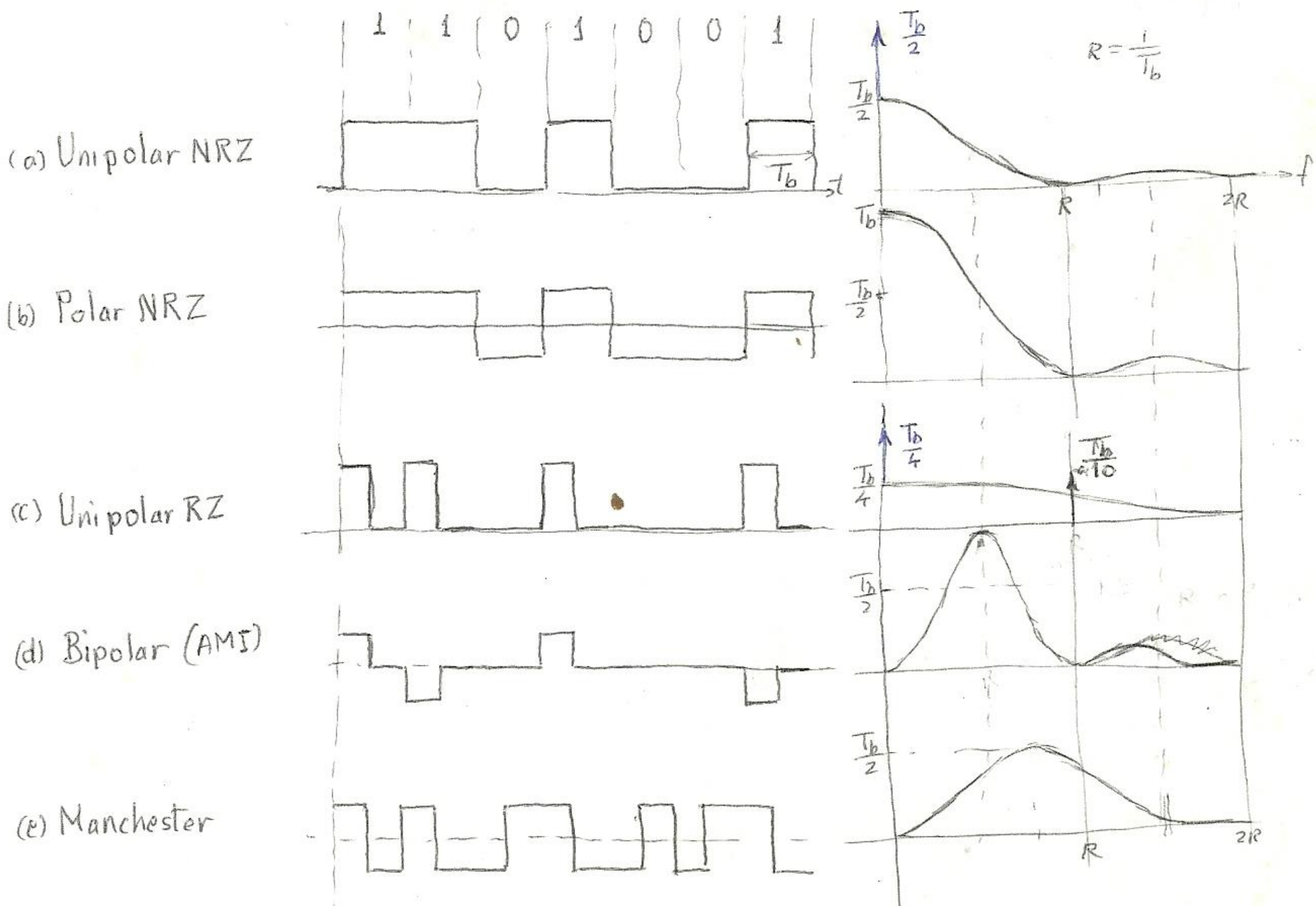
- **Bipolar**:
- **The binary 1 is represented by alternatively positive or negative values over a half-bit period.**
- **The binary 0 is represented by a zero level.**

Manchester

- Split phase encoding:
- Each binary 1 is represented by a positive half-bit period pulse followed by a negative half-bit period pulse.
- Similarly, a binary 0 is represented by a negative half-bit period followed by a positive half-bit period.

Power Spectra and Comparison

Power Spectra and Comparison



Unipolar and Polar Signaling NRZ

- **Advantage:** They are easy to generate (TTL and CMOS circuits).
- **Disadvantage:** Waste of power due to the dc level so that dc coupled circuits are needed.
- Polar requires one supply although unipolar requires positive and negative supplies.
- Polar signaling has a superior bit error performance than other methods.

Unipolar RZ

- In addition to the continuous power spectrum.
- It has discrete spectral lines at the odd multiples of the bit rate.
- It requires twice bandwidth of that of the NRZ codes.

Bipolar

- Neither DC nor discrete spectra's.
- Bandwidth is that of NRZ.

Manchester

- Neither DC nor discrete lines.
- Bandwidth is twice that of NRZ.

Spectral Efficiency

- Number of bits per second of data that can be supported by each hertz of bandwidth.

$$\eta = \frac{R}{B} \qquad (b/s/Hz)$$

- Unipolar, polar, and bipolar signals are twice as efficient as RZ or Manchester signals?
- Much greater efficiency can be achieved with multilevel signaling.
- Theoretical limit is determined by Shannon's?

$$\eta = \frac{C}{B} = Log\left[1 + \frac{S}{N}\right]$$

Why Differential

- When serial data are passed through many circuits, waveform is often inverted (i.e., data complemented).
- This happen in twisted-pair channel just by switching the two leads at a connection point when polar signaling is used.
- For this reason differential coding is often employed

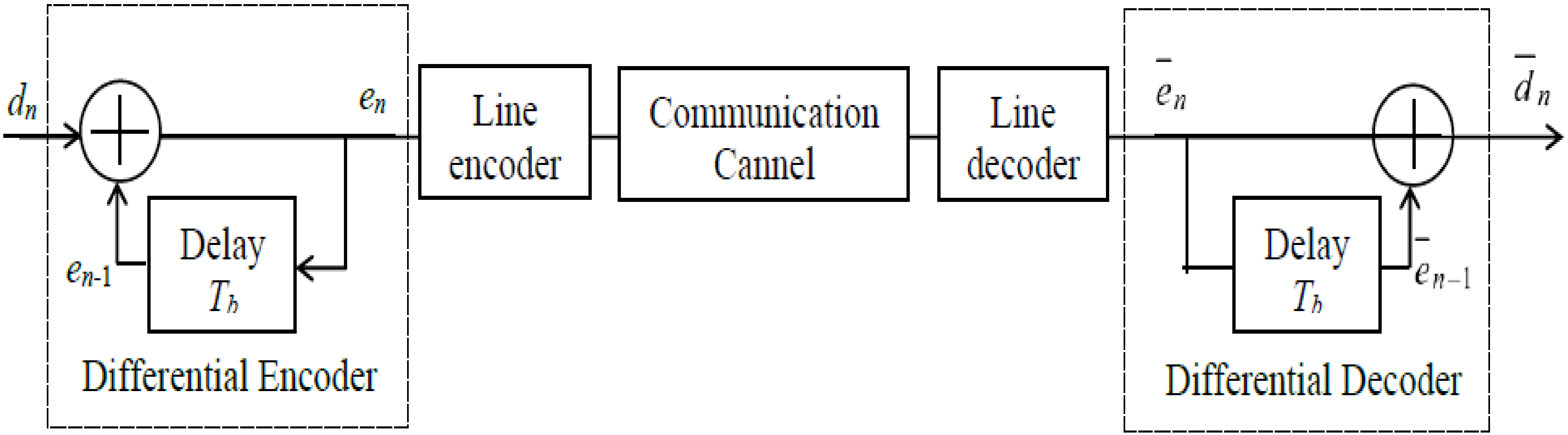
Differential Encoding

- Each digit is obtained by comparing the present input bit with the past encoded bit.
  - Binary 1 is encoded if the present bit and past encoded bit are of opposite state, and
  - Binary 0 is encoded if the states are the same.
- Encoded differential data are generated by:

$$e_n = d_n \oplus e_{n-1}$$

$\oplus$ modulo 2 adder, or an exclusive OR gate

Differential Coding System



Differential Decoding

- The received decoded data are given by:

$$\hat{d}_n = \hat{e}_n \oplus \hat{e}_{n-1}$$

$\oplus$ modulo 2 adder, or an exclusive OR gate

Synchronous and Asynchronous Lines

Synchronous System

- Each device is designed so that internal clock is relatively stable for a long period of time,
- And it is synchronized to a system master clock.
- Therefore, each bit of data is clocked in synchronism with the master clock.
- It requires a higher levels of synchronization to allow the receiver to determine the beginning and end of each block of data.
- Synchronizing signal may be:
  - Provided by a separate clocking line, or
  - Embedded in data signal (using Manchester)

Example

US telephone industry plans to synchronize all of the digital lines with a master clock located in Hillsboro.

Asynchronous System

- The timing is precise only for the bits within each character (or word).
- It is called start-stop signaling because each character consists of a start bit that starts the receiver clock and concludes with one or two stop bits that terminate the clocking.
- The receiver clock is started aperiodically.
- So, no synchronization with a master clock is required.

Example

- Keyboard terminals are asynchronous sources.
- The complete character consists of 10 bits:
  - One start bit,
  - 7-bit of the ASCII code,
  - One parity bit, and
  - One stop bit (for R>300 bits/sec).
- In asynchronous TDM, different sources are multiplexed on a character-interleaved basis instead of interleaving on a bit-by-bit basis.

Multiplexers Classified

- TDM to synchronous lines.
- TDM to quasi-synchronous lines:
  - Individual clocks of the input data sources are not exactly synchronized in frequency.
  - So there will be some variation in the bit rates from different sources.
  - Sometimes, the clock rates are not related by a rational number.
  - This requires stuff bits, which are dummy bits; 1's, 0's, or some alternating pattern, to compensate for the difference sources rates.
- TDM to asynchronous lines: It produces:
  - High-speed asynchronous output (no stuff bits required).
  - High-speed synchronous output (staff bits is required).